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1996 J. Phys. A: Math. Gen. 29 L351

(http://iopscience.iop.org/0305-4470/29/13/007)

J. Phys. A: Math. Gen. **29** (1996) L351–L354. Printed in the UK

LETTER TO THE EDITOR

Amplification by stochastic interference

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Received 13 December 1995, in final form 12 April 1996

Abstract. A new method is introduced to obtain a strong signal by the interference of weak signals in noisy channels. The method is based on the interference of $1/f$ noise from parallel channels. One realization of stochastic interference is the auditory nervous system. Stochastic interference may have broad potential applications in information transmission by parallel noisy channels.

The method of stochastic interference was originally conceived for information processing in the auditory nervous system [1]. It makes use of the random fractal geometry of the spike discharge patterns [2–5] which are processed by diverging and converging information networks of the auditory system. This method is distinct from stochastic resonance [6], but when both methods are combined, a fascinating new model of transsynaptic information transfer emerges [7].

Here, we are interested in more general aspects of stochastic interference. The method can be sketched as follows. Consider an information transmission via multiple channels. Assume further that the information is coded in statistically self-similar, random [8–14] fractal patterns [15]. The idea that information is encoded in the dimensional geometry of random fractals is not entirely new [2–5]. But here, n fractal information signals (with the same dimensional parameter) are combined by logical 'and' operations (equivalent to the set theoretic intersection) to form a new signal. The new signal also has a fractal geometry. Its fractal dimension varies n times as strongly as the variations of the dimensional parameter of the primary signal. Thus, when multiple information channels are combined properly, arbitrary weak variations of their input signals can be amplified to arbitrary strong variations of the resulting output channel.

Stochastic interference operates with $1/f^\beta$ noise [16, 17], characterized by a power spectral density of $S_V(f) \propto 1/f^\beta$. This noise corresponds to a signal $X(t)$ at time t whose graph $\{(t, X(t)) \mid t_{\rm min} \leqslant t \leqslant t_{\rm max}\}$ has a random fractal geometry. The fractal (box-counting) dimension of the graph can be approximated by [18, 19]

$$D = \min\left\{2, E + \frac{3-\beta}{2}\right\} \tag{1}$$

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where E is the (integer) dimension of the noise. For one-dimensional noise, $E = 1$. White noise corresponds to $\beta = 0$, brown noise corresponds to $\beta = 2$, whereas systems showing $1/f$ noise operate at approximately $\beta = 0.8$–1.2.

Consider a sequence of zeros and ones which constitutes a fractal pattern. Such a random fractal of dimension D can, for instance, be recursively generated starting with a sequence of ones. First, the sequence is subdivided into k blocks of sequences of length δ symbols. Then, a fraction of $1 - \exp[(D-1)\log(k)]$ blocks of length δ symbols is filled with zeros (instead of ones). Next, one takes the remaining pieces of the pattern containing ones and repeats the same procedure (the length of the blocks decreases by a factor of k, until one arrives at $\delta = 1$) [19].

The fractal dimension of a random fractal signal can be understood as follows. Divide a sequence of zeros and ones again into k blocks of length δ. Count how many of these blocks contain ones at all (or, more realistically for practical applications, up to a density s). If r is the number of filled blocks, then the fractal (box-counting) dimension is given by

$$D = \frac{\log r}{\log(1/\delta)} \tag{2}$$

independently of the scale resolution δ. The fractal dimensional measure D should be robust with respect to variations in methods of determining it. That is, it should remain the same, regardless of the method by which it is inferred.

Information can be encoded by the random fractal patterns of $1/f$ noise, in particular by variations of the dimension parameter. More precisely, assume, for example, two source symbols s_1 and s_2 are encoded by (RFP stands for 'random fractal pattern')

$$\#(s_i) = \begin{cases} \text{RFP with } 0 \leqslant D(\text{RFP}) < D_\mathrm{c} & \text{if } s_i = s_1 \\ \text{RFP with } D_\mathrm{c} \leqslant D(\text{RFP}) \leqslant E & \text{if } s_i = s_2 \end{cases} \tag{3}$$

where D_c is a 'critical dimension parameter'.

As has been pointed out by Falconer [19], under certain 'mild side conditions', the intersection of two random fractals A_1 and A_2 which can be minimally embedded in $\mathbb{R}^E$ is again a random fractal with dimension

$$D(A_1 \cap A_2) = \max\{0, D(A_1) + D(A_2) - E\}. \tag{4}$$

By induction, (4) generalizes to the intersection of an arbitrary number of random fractal sets. Thus, the dimension of the intersection of n random fractals $\mathcal{A} = \{A_1, \ldots, A_n\}$ is given by

$$D(\mathcal{A}) = D\left(\bigcap_{i=1}^{n} A_i\right) = \max\left\{0, -E(n-1) + \sum_{i=1}^{n} D(A_i)\right\}. \tag{5}$$

We shall concentrate on the case of one-dimensional signals where $E = 1$. Assume that the signals are represented by sequences of zeros and ones. Assume further that we have n random fractal signals $A_1, \ldots, A_n$. Each of these sequences is transmitted in a separate channel. The sequences are then recombined to form a new, secondary signal sequence. In particular, we shall be interested in the *intersection* of n signals encoded by random fractal patterns. An intersection of two signals $A_1 = a_{11}a_{12}a_{13}\ldots a_{1m}$ and $A_2 = a_{21}a_{22}a_{23}\ldots a_{2m}$ of length m, $a_{ij} \in \{0, 1\}$, is again a signal $A_1 \cap A_2 = A_3 = a_{31}a_{32}a_{33}\ldots a_{3m}$ of length m which is defined by

$$a_{3i} = \begin{cases} 1 & \text{if} \quad a_{1i}a_{2i} = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

We shall denote this arrangement by the term *stochastic interference*. Taking the product in (6) amounts to the logical 'and' operation, if 0 and 1 are identified with the logical values 'false' and 'true', respectively.

Let us briefly discuss two features of *stochastic interference*. First, the combination of white noise, denoted by $\mathbb{I}$ with $D(\mathbb{I}) = 1$, with a random fractal signal A, results in the recovery of the original fractal signal with the original dimension; i.e. (5) reduces to

$$D(A \cap \mathbb{I}) = D(A) + D(\mathbb{I}) - 1 = D(A). \tag{7}$$

Stated pointedly: apart from a reduction in intensity, white noise does not affect the coding.

Second, by assuming that all n random fractals have equal dimensions, i.e. $D(A_i) = D$ for $1 \leqslant i \leqslant n$, (5) reduces to

$$D(\mathcal{A}) = \max\{0, n(D-1)+1\}. \tag{8}$$

An immediate consequence of (8) is that, for truly fractal signals ($D < 1$), any variation of the fractal dimension of the secondary signal is directly proportional to the number, n, of the primary signals; i.e.

$$\Delta D(\mathcal{A}) = n\Delta D \quad \text{for} \quad D \neq 1. \tag{9}$$

Therefore, the more channels there are, the more the dimension of the secondary source varies in response to variations of the primary source; there is an 'amplification' of any change in the primary signal. Figure 1 shows the results of a computer experiment.

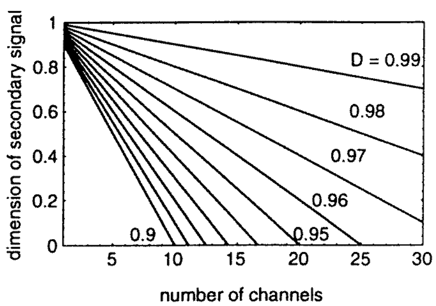

Figure 1. Theoretical prediction of $D^{\cap}(n)$ versus n for various values of the dimension D.

This amplification, however, has a price: any increase in the amplification of the variation of the primary dimension obtained by additional channels results in a reduction of the overall secondary signal strength.

In figure 2, the number of critical channels, for which the secondary signal vanishes (all $a_{3i} = 0$), is drawn against the dimension of the primary signals. One arrives at the number of critical channels n_c by setting $D(\mathcal{A}) = 0$ in (8) and solving for n; that is:

$$n_c = \frac{1}{1-D} \quad \text{for } 0 \leqslant D < 1. \tag{10}$$

For a channel number in the range 10–20, the fractal dimension of the primary signal has to lie in the range 0.9–1 in order to balance the attenuation.

We close this short discussion of stochastic interference by pointing out the possibility of a twofold information transfer in one and the same system of multiple noisy channels: first, transfer by the standard coding techniques [20], and second, modulated by it, transfer

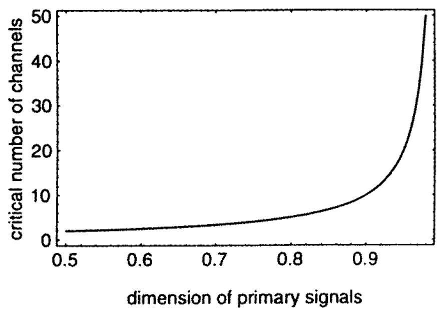

Figure 2. Theoretical prediction of the critical number of channels as a function of the dimension of the primary signal.

by information coding using $1/f$ noise with stochastic interference. This form of double-band information transfer may be realized in the auditory pathway of mammals and also has potential applications in communication technology.

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